Homework 2

Inductive Theorem Proving

February 13, 2007

Generic hints:

- understand the specifications before starting the proof;
- use induction and auto. they work in most of the cases;
- most of the lemmas you will need involve properties of natural numbers such as associativity or distributivity;
- ITP does not know to automatically do case analysis; sometimes you might need to request a case analysis be performed by splitting on a boolean condition.
- again, let the specification guide your proof;

Exercise 1 *Prove that for any N,*

$$1 + 2 + \ldots + N = \frac{N(N+1)}{2},$$

or, equivalently, that $(1+2+\ldots+N)*2 = N*(N+1)$. Use the specification provided in file `1-nat-sum.itp`.

Exercise 2 *Using the specification of multi-sets (i.e., sets with repetition) provided in file* `2-card.itp`*, prove that the number of elements in the union of two multi-sets is the sum of the number of elements in each multi-set.*

Exercise 3 *Using the specification in file* `3-nat-sum2.itp`*, prove that for any N,*

$$1^2 + 2^2 + \ldots + N^2 = \frac{N(N+1)(2N+1)}{6},$$

or, equivalently, that $(1^2 + 2^2 + \ldots + N^2) * 6 = N * (N+1) * (2N+1)$.

Hint: *you might need to prove distributivity of multiplication over addition for naturals.*

Exercise 4 *Prove that the specification of the insertion-sorting algorithm described in the lecture notes does not alter the contents of the list being sorted. Use the specification in file* `4-isort.itp`

Exercise 5 ***(extra-credit)*** *In the context of the exercise above, prove that the list obtained after applying the insertion algorithm is sorted.*